EVALUATION OF ELASTIC PROPERTIES OF SURFACE LAYER FROM RAYLEIGH WAVE DISPERSION

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INTRODUCTION

This paper deals with the problem of determining subsurface elastic properties in non homogeneous layers from the measured dispersion of Rayleigh surface waves. The thermochemical treatments of steels such as carburizing or nitrurizing, produce a gradient of properties in the depth which is caused by carbon or nitrogen diffusion. These treatments are usually performed in the manufacture of automotive components (gear box, c.v. joint, hooke joint).

The use of surface Rayleigh waves allows information to be obtained in the depth of the material by changing penetration with frequency. By using signal processing in the frequency domain, we may achieve the dispersion curve of velocity which is a function of the gradient of properties of the layer.

The profile of elastic properties should be calculated from the measured dispersion of velocity data. This situation requires the solution of the inverse problem, which has been solved by using a perturbation integral equation that describes the influence of the gradient.

THEORETICAL BACKGROUND

In the case of non homogeneous layers, the density ρ of the material and the elastic properties C_{ij} are functions of the position in the depth. The mathematical problem cannot be approached by solving the equation of wave motion which doesn't propose any analytical solution in the general case. Many authors [1-4] have dealt with this problem and proposed numerical methods.

Roznowski [5] proposed to simplify the problem in the case of small nonhomogeneity in isotropic non homogeneous layer by assuming that the shear modulus $\mu=\mu(z)$ is a function of the depth, whereas mass density ρ and Poisson ratio ν are constant. The author solves a two dimensional equation of motion in the isotropic non homogeneous medium. In order to obtain a closed form solution for this problem, he shall specify the function $\mu(z)$.

An other way to deal with the problem was proposed by Szabo [6], Tittman and Richardson [7-8]. They used a perturbation theory integral equation that describes the influence of a known gradient on the velocity dispersion. A small change in the material depth (mass density, compliance) is represented by the gradient function F(z).

INVERSE SOLUTION

Tittman and Thompson [1] have applied the equation derived by Auld [9] to non-destructive evaluation of subsurface gradients. For a SAW traveling along the +x axis to a semi-infinite isotropic solid with the +z axis directed in the depth, they have shown that Auld's equation can be written in the form :

$$\frac{\Delta V_R}{V_R} = \int_0^{\infty} \left\{ M_1 \exp(-2\alpha_1 z/\lambda) + M_2 \exp[-(\alpha_1 + \alpha_2) z/\lambda] + M_3 \exp[-2\alpha_2 z/\lambda) \right\} F(z)\, dz/\lambda \tag{1}$$

where : $\lambda=V_R/f$, wavelength - M_1, M_2, M_3, α_1, α_2 : constants calculated from the Auld's equation

Equation (1) allows to calculate the dispersion of velocity of SAW for a known gradient.

To solve the inverse problem, we must find the gradient function F(z) from the measured dispersion of velocity data. The equation described by Tittmann and Thomson [1] may be recast in the following form :

$$\frac{\Delta V_R(f)}{f} = \int_0^{\infty} \left(\sum_{i=1}^{3} M_i \exp[-q_i f . z] \right) F(z) dz \tag{2}$$

Each term in the equation (2) can be recognized as a scaled Laplace transform defined as :

$$\overline{F}(q_i f) = L(F(z)) = \int_0^{\infty} \exp[-q_i \cdot f \cdot z] F(z) dz \tag{3}$$

Equation (2) may be rewritten as :

$$\frac{\Delta V_R}{f} = \sum_{i=1}^{3} M_i \overline{F}(q_i f) \tag{4}$$

By taking the inverse Laplace transform of equation (4) we get :

$$L^{-1}\left[\frac{\Delta V_R}{f}\right] = \sum_{i=1}^{3} \frac{M_i}{q_i} F\left(\frac{z}{q_i}\right) \quad (5)$$

The M_i and q_i values are known constants so that if we measure the dispersion of velocity, it is possible to determine the gradient function F(z).

By assuming that the function F(z) and the dispersion of velocity of the SAW can be represented by polynominal functions which have the same order, Szabo proposed the following form :

$$F(z) = \sum_{n=0}^{N} b_n Z^n \quad (6)$$

Substitution of equation (6) in equation (5) yields :

$$L^{-1}\left[\frac{\Delta V_R}{f}\right] = \sum_{n=0}^{N} \left(\frac{M_1}{q_1^{n+1}} + \frac{M_2}{q_2^{n+1}} + \frac{M_3}{q_3^{n+1}} \right) b_n z^n \quad (7)$$

From the experimental dispersion of velocity data, it is possible to fit a curve by using the least-squares fit method :

$$\Delta V_R(\lambda) = \sum_{n=0}^{N} a_n \lambda^n \quad \text{or} \quad \frac{\Delta V_R(f)}{f} = \sum_{n=0}^{N} a_n \frac{V_R^n}{f^{n+1}} \quad (8)$$

By taking the inverse transform of equation (8), we obtain :

$$L^{-1}\left[\frac{\Delta V_R}{f}\right] = \sum_{n=0}^{N} a_n \frac{V_R^n Z^n}{n!} \quad (9)$$

Finally the series of eqs. (7) and (9) may be equated term by term to give an expression for b_n :

$$b_n = \frac{a_n V_R^n}{n!} x \left\{ \frac{M_1}{q_1^{n+1}} + \frac{M_2}{q_2^{n+1}} + \frac{M_3}{q_3^{n+1}} \right\}^{-1} \quad (10)$$

The function $F(z) = \sum_{n}^{N} b_n z^n$ can be determined, the b_n coefficients of the polynominal expression F(z) are finally calculated from the a_n coefficients of the fitting dispersion data curve.

ULTRASONIC TECHNIQUE

The experimental measurements are made with double wedge transducers using refraction technique. The piezoelectric transducer is mounted near the critical angle corresponding to surface Rayleigh waves. These transducers have a wide bandwidth (1-10 Mhz) which allows to achieve the dispersion curve of velocity by using signal processing. The transducer wave length range (0,3 to 3 mm) is suitable to the case of carburized steels, which presents case depths between 1 and 2 mm.

The dispersion velocity curve is measured from the variation of time of flight between a reference state (non perturbed material) and the perturbed material with carbon diffusion layer. The phase of the cross-spectrum function of the two signals gives the incremental time for each frequency enclosed in the bandwith of the transducer.

Table 1 : Physical constants used for calculation (mean values for non carburized state)

	E GPa	ν	ρ kg/m³
27 MnCr5	209,9	0,286	7816
16 NiCr6	209	0,29	7840
	± 1	± 0,05	± 10

Each experimental dispersion curve of velocity is determined from twenty five points (Δf=500 Khz). Because the increment in wavelength is not linear ($\Delta\lambda$=VR/fn , fn = 0.5, 1, ... 12 Mhz) we obtain more data for the lowest wavelengths. To improve the least-squares fit of the dispersion curve, we added (20) points in the area corresponding to the lowest wavelenth (1 to 3 mm) in order to have a constant increment in wavelength.

The exploitation of results will be made as Young's Modulus profiles. The Young's modulus E may be directly calculated from the Lame's constants λ et μ which are defined by the function F(z).

EXPERIMENTAL RESULTS

The experimental results are concerned with the characterization of carburized steels. The case depth for the different treatments vary from 1 to 2 mm. Samples (∅ = 40 mm, thickness = 10 mm) were machined from bars in 27 MnCr5 and 16 NiCr6 steels. (Mechanical characterization of carburized steels is made by Hv1 hardness profiles). The mean values of Young's modulus, Poisson ratio and density for the used steels are listed in table 1.

Before ultrasonic characterization of carburized steels, we made profile of carbon diffusion in the different layers as in Fig. 1A. The determination of case depth is achieved by hardness profile (Vickers test) in the depth of the layer. The case depth corresponds to the depth in the material for which the hardness value is equal to 550 Hv (NF A 04.202 standard). The Fig. 1B presents the hardness profiles for three carburized layers (0.6, 0.92, 1.6 mm).

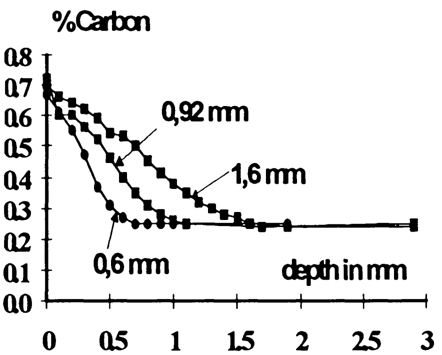

Fig.1 A

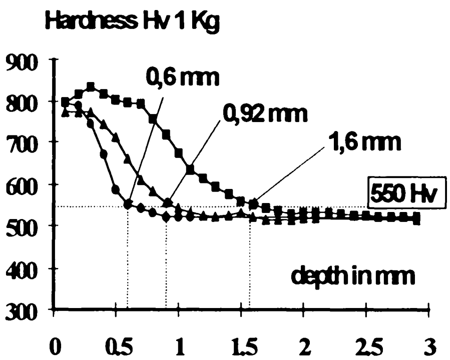

Fig.1 B

Figure 1 : Carbon diffusion and hardness profiles of carburized layers : usual case

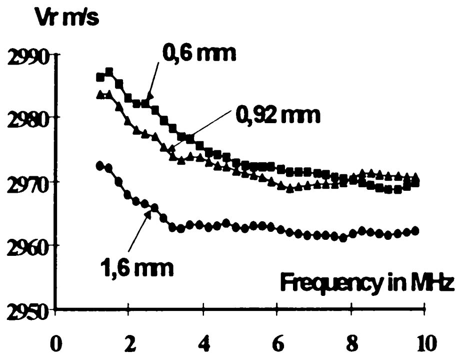

Fig.2 A

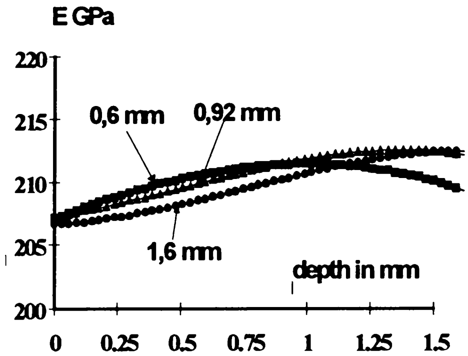

Fig.2 B

Figure 2 : Characterization of carburized layers : usual case

The dispersion of velocity for Rayleigh surface wave and the Young's modulus profiles are presented on the Fig. 2A and 2B. The dispersion velocity curve is done by using signal processing as we explain previously. The Young's modulus profile is calculated from the Szabo's model, by the least-mean squares fitting of the dispersion curve $\frac{\Delta V_R(f)}{f}$ (exp. 9 and 10).

NON USUALLY CARBURIZED LAYERS

This part presents results obtained during wrong heat treatment. In the case of carburized steel, after the diffusion process, the material is quenched. When the carbon content is high at the surface (> .8%), the quench may activate the apparition of austenic phase or carbides (Fe_3C). The Fig. 3A and 3B show the influence of such a situation combined with a wrong quenching temperature.

This treatment induces a slighty evolution on the dispersion curve of Rayleigh wave velocity (Fig. 4A) and a particular Young's modulus profile (Fig. 4B).

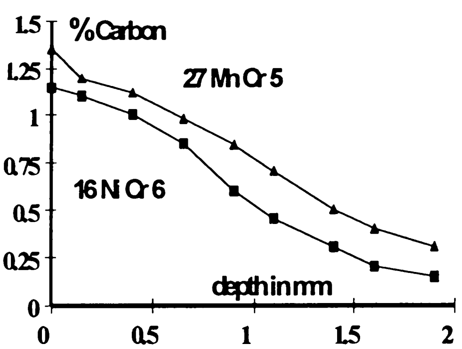

Fig.3 A

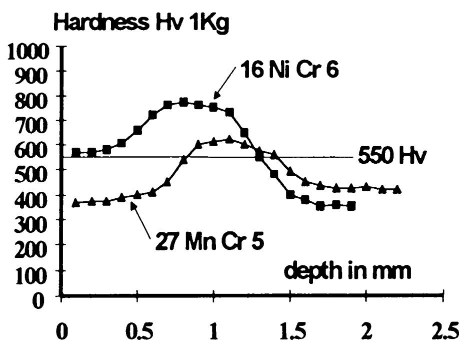

Fig. 3 B

Figure 3 : Carbon diffusion and hardness profiles of carburized layers : non usual treatment

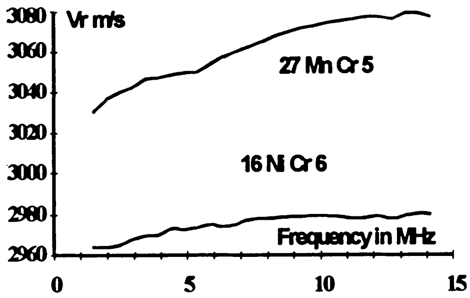

Fig.4A

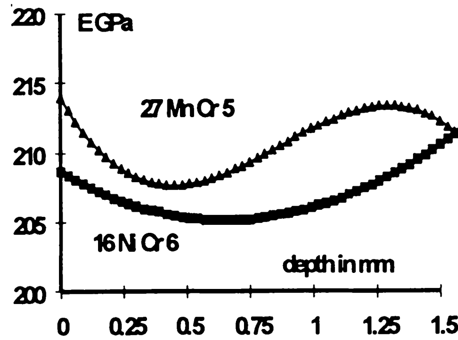

Fig. 4B

Figure 4 : Characterization of carburized layers : non usual treatment

DISCUSSION

The statistical analysis by least-mean-squares fit of the dispersion curve $\Delta v(\lambda) = P_n(\lambda) = \sum_0^n a_k \lambda^k$ allows calculation of the F(z) function. For the carburized layers studied, the least-quares fit was made from the first to the sixth power. The degree of the polynominal regression giving the best fit may be determined by the calculation of the variance estimation $V[P_k(\lambda)]$. Best fit was obtained for degrees higher than four.

For the F(z) function we apply the same argument and find that only polynominal regression with degree lower than 4 could minimize the variance V[F(z)] of F(z). That comes from the b_n coefficients of the polynominal expression of F(z) which are adjusted by q_i and M_i constants which only depend on the non perturbed material. For this study, the F(z) function was finally defined as the mean value of the $F_k(z)$, (k : 1 to 3) where k is the degree of the polynominal regression of $\Delta V(\lambda)$.

The dispersion curves of velocity presented in the Fig. 2 allow to discriminate the three case depths (0.6, 0.92 and 1.6 mm). As shown on the carbon diffusion profile (Fig. 1A) and hardness profile (Fig. 1B), both are closed, and we may observe the same behaviour on the velocity dispersion curve (Fig. 2A). The Young's modulus E profile (Fig. 2B) shows a smooth evolution between the near surface and the substrate. In the near surface, the E magnitude is about 207 GPa and reaches 210 GPa in the substrate. These results show good agreement with data measured on bulk heat treated steels (% C $\leq$ .8).

For the case of carburized layers in Fig. 3, the anomaly of heat treatment is due to a wrong quenching temperature associated with a high carbon content at the surface (% C > 1), which leads to (Fe_3C) carbides formation. This situation involves a particular hardness profile as shown on the Fig. 3B which presents a diminution of the hardness magnitude near the surface (0 to 0.5 mm). The layer is correctly heat treated from 0.5 mm in depth to the substrate. The Young's modulus profile (Fig. 4B) is the reverse of the hardness profile.

Table 2 : Physical constants calculated in function of the depth

	E(GPa) - depth =0mm	E(GPa) - depth=0,5mm	E(GPa) - depth=1.5mm
27 MnCr5	214	207	209
16 NiCr6	209	205	209

This behaviour may be explained by the fact that Young's modulus corresponding to martensite has a lower value than the ferritic phase one's. Table 2 shows the different values obtained in the depth for the two layers.

CONCLUSION

The dispersion curves of Rayleigh waves velocity give an interesting approach to characterize superficial treatments which causes non homogeneity in the material. For the case of carburized steels, the Young's modulus profiles calculated from Szabo's model allowed to verify that the E values near the surface and below the carbon diffusion layer (depth $\geq$ 1.5 mm) are well correlated with experimental data. These data were obtained in bulk homogeneous heat treated specimens giving identical properties (hardness). Unfortunately, it was not possible to verify the profile evolution of Young's modulus with another experimental technique.

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